MATH 105A and 110A Review: Eigenvalues and diagonalization

Facts to Know:

Let A be any $n \times n$ matrix. A nonzero vector x is said to be an **eigenvector** of A if

The λ above is called an

λ is an eigenvalue of A if and only if

has a nontrivial solution. We find eigenvalues by solving the **characteristic polynomial**:

We say $\lambda = a$ is an eigenvalue of A with **multiplicity** k if

where

A matrix D is said to be a **diagonal** matrix if:

A matrix A is said to be **diagonalizable** if there exists a diagonal matrix D and some invertible matrix such that

Any $n \times n$ matrix A is diagonalizable if and only if A has

The diagonalization of A:

Any $n \times n$ matrix A is diagonalizable if and only if the following two hold:

1. All the eigenvalues are
2. If λ is an eigenvalue of multiplicity k, then there are

Examples:

1. Diagonalize the following matrix if possible:

$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$$